6.2 Solving Linear Systems by Addition

Solving a system by graphing is limited by the accuracy of the graph. When the intersecting point is not an exact integer, it is difficult to determine the coordinates from the graph. Solving a system algebraically does not depend on a graph, and always gives accurate coordinates. The method used to solve a system algebraically is an extension of the addition property used to solve an equation for a single variable.

Rule of Addition

For all real numbers a, b, c and d: if $a = b$, and $c = d$, then $a + c = b + d$

Solving a Linear System by the Addition Method

1. Write the equations of the system in general form $ax + by = c$.
2. Multiply the terms of one equation, or both of the equations, by a constant such that the coefficients of x or y are different only in their sign.
3. Add the equations, and solve the resulting equation.
4. Substitute the value obtained in step 3 into either of the original equations, and solve for the remaining variable.
5. Steps 3 and 4 give the solution to the system.
6. To check the solution take the values from steps 3 and 4, and substitute them into the equation not used in step 4.

Example 1 Solve
$$\begin{aligned} x + y &= 6 \\ x - y &= 4 \end{aligned}$$

► *Solution*:
$$\begin{array}{l} x + y = 6 \\ \underline{x - y = 4} \\ 2x \quad = 10 \quad \rightarrow \quad x = 5 \end{array}$$

To find y, substitute $x = 5$ in one of the original equations.

$x + y = 6 \rightarrow 5 + y = 6 \rightarrow y = 1$

Check: Substitute (5, 1) into the equation not used, $x - y = 4 \rightarrow 5 - 1 = 4 \rightarrow 4 = 4$. (True)

The solution to the system is (5, 1).

Example 2 Solve $y = x + 4$
$x + 2y = 5$

► *Solution*: Rewrite $y = x + 4$ as $-x + y = 4$.

$$\begin{array}{r} -x + y = 4 \\ x + 2y = 5 \\ \hline 3y = 9 \end{array} \quad \rightarrow \quad y = 3$$

To find *x*, substitute $y = 3$ in one of the original equations.

$x + 2y = 5 \rightarrow x + 2(3) = 5 \rightarrow x = -1$

Check: Substitute (− 1, 3) in $y = x + 4 \rightarrow 3 = -1 + 4 \rightarrow -3 = -3$. (True)

The solution to the system is (− 1, 3).

Example 3 Solve $2x - 3y = 2$
$x + 2y = 8$

► *Solution*: To obtain coefficients for *x* that differ only in sign, multiply the second equation by −2. Add the results to obtain an equation that has only one variable *y*.

$$\begin{array}{r} 2x - 3y = 2 \\ -2(x + 2y = 8) \end{array} \quad \rightarrow \quad \begin{array}{r} 2x - 3y = 2 \\ -2x - 4y = -16 \\ \hline -7y = -14 \end{array} \quad \rightarrow \quad y = 2$$

To find *x*, substitute $y = 2$ in one of the original equations.

$x + 2y = 8 \rightarrow x + 2(2) = 8 \rightarrow x = 4$

Check: Substitute (4, 2) in $2x - 3y = 2 \rightarrow 2(4) - 3(2) = 2 \rightarrow 2 = 2$. (True)

The solution to the system is (4, 2).

Example 4 Solve $4x + 3y = 5$
$3x - 2y = 8$

► *Solution*: Multiply equation 1 by 2 and equation 2 by 3, then add the results.

$$\begin{array}{r} 2(4x + 3y = 5) \\ 3(3x - 2y = 8) \end{array} \quad \rightarrow \quad \begin{array}{r} 8x + 6y = 10 \\ 9x - 6y = 24 \\ \hline 17x \quad\quad = 34 \end{array} \quad \rightarrow \quad x = 2$$

To find *y*, substitute $x = 2$ in one of the original equations.

$4x + 3y = 5 \rightarrow 4(2) + 3y = 5 \rightarrow 3y = -3 \rightarrow y = -1$

Check: Substitute (2, − 1) in $3x - 2y = 8 \rightarrow 3(2) - 2(-1) = 8 \rightarrow 8 = 8$. (True)

The solution to the system is (2, − 1).

Example 5 Solve $\frac{x}{2} - \frac{y}{3} = -\frac{7}{12}$

$\frac{x}{8} + \frac{y}{9} = 0$

► *Solution*: Multiply equation 2 by 3, then add the results.

$$\begin{array}{l} \frac{x}{2} - \frac{y}{3} = -\frac{7}{12} \\ 3\left(\frac{x}{8} + \frac{y}{9} = 0\right) \end{array} \rightarrow \begin{array}{l} \frac{x}{2} - \frac{y}{3} = -\frac{7}{12} \\ \frac{3x}{8} + \frac{y}{3} = 0 \\ \hline \frac{x}{2} + \frac{3x}{8} = -\frac{7}{12} \end{array} \rightarrow \begin{array}{r} 24\left(\frac{x}{2} + \frac{3x}{8} = -\frac{7}{12}\right) \\ 12x + 9x = -14 \\ 21x = -14 \\ x = -\frac{14}{21} = -\frac{2}{3} \end{array}$$

To find y, substitute the value of $-\frac{2}{3}$ for x in one of the original equations.

$$\frac{x}{8} + \frac{y}{9} = 0 \rightarrow \frac{(-2/3)}{8} + \frac{y}{9} = 0 \rightarrow -\frac{1}{12} + \frac{y}{9} = 0 \rightarrow \frac{y}{9} = \frac{1}{12} \rightarrow y = \frac{3}{4}$$

Check: Substitute $\left(-\frac{2}{3}, \frac{3}{4}\right)$ in $\frac{x}{2} - \frac{y}{3} = -\frac{7}{12} \rightarrow \frac{1}{2}\left(-\frac{2}{3}\right) - \frac{1}{3}\left(\frac{3}{4}\right) = -\frac{7}{12}$. (True)

The solution to the system is $\left(-\frac{2}{3}, \frac{3}{4}\right)$.

Example 6 Solve $3x - 2y = 1$

$-6x + 4y = 3$

► *Solution*: Multiply equation one by 2, then add the results.

$$\begin{array}{l} 2(3x - 2y = 1) \\ -6x + 4y = 3 \end{array} \rightarrow \begin{array}{r} 6x - 4y = 2 \\ -6x + 4y = 3 \\ \hline 0 = 5 \end{array}$$

There is no solution, so the lines must be parallel.

Example 7 Solve $2x + 5y = 2$

$-4x - 10y = -4$

► *Solution*: Multiply equation one by 2, then add the results.

$$\begin{array}{l} 2(2x + 5y = 2) \\ -4x - 10y = -4 \end{array} \rightarrow \begin{array}{r} 4x + 10y = 4 \\ -4x - 10y = -4 \\ \hline 0 = 0 \end{array}$$

There are infinite solutions, so the lines must coincide.